UGEB2530 Game and strategic thinking
Solution to Assignment 1

Due:26 Jan 2014 (Monday)

1. Find all pure Nash equilibria of the following games.

   (a) $\begin{pmatrix} (4,-4) & (1,-2) \\ (3,5) & (-2,7) \end{pmatrix}$

   (b) $\begin{pmatrix} (5,3) & (1,-2) \\ (3,0) & (4,5) \end{pmatrix}$

   Solution:

   (a) Both players have their own dominant strategy, and the pure Nash equilibrium is $(1,-2)$;

   (b) Using the definition of Nash equilibrium, $(5,3)$ and $(4,5)$ are the pure Nash equilibria.

2. There is a 4-face dice and the numbers on the 4 faces are 1,1,2 and 3 respectively. The dice is thrown once.

   (a) Find the expected value of the number at the bottom.

   (b) Find the expected value of the square of the number at the bottom.

   Solution:

   (a) Let X be the random variable standing for the number at the bottom, $E(X) = 1 \times \frac{1}{2} + 2 \times \frac{1}{4} + 3 \times \frac{1}{4} = \frac{7}{4}$;

   (b) Let Y be the random variable standing for the square of number at the bottom, $E(Y) = 1 \times \frac{1}{2} + 4 \times \frac{1}{4} + 9 \times \frac{1}{4} = \frac{15}{4}$.

3. In a Rock-Paper-Scissors game, the loser pays the total number of fingers in the two gesture to the winner. The payoffs of the players are 0 if there is a draw.

   (a) Write down the game matrix (payoff of player 1) of the game. (Use Rock, Paper, Scissors, as the order of strategies.)

   (b) Suppose player 1 uses $(0.2, 0.3, 0.5)$ and player 2 uses $(0.3, 0.4, 0.3)$. Find that expected payoff of player 1.

   (c) If player 1 uses $(0.2, 0.3, 0.5)$, what is the best strategy of player 2.

   (d) If player 2 uses $(0.3, 0.4, 0.3)$, what is the best strategy of player 1.

   Solution:

   (a) The game matrix is shown below:

	Rock	*Paper*	*Scissors*
Rock	0	-5	2
Paper	5	0	-7
Scissors	-2	7	0

(b) The expected payoff is calculated as:

$$\begin{bmatrix} 0.2 & 0.3 & 0.5 \end{bmatrix} \begin{bmatrix} 0 & -5 & 2 \\ 5 & 0 & -7 \\ -2 & 7 & 0 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.4 \\ 0.3 \end{bmatrix} = 0.64.$$

(c)

$$\begin{bmatrix} 0.2 & 0.3 & 0.5 \end{bmatrix} \begin{bmatrix} 0 & -5 & 2 \\ 5 & 0 & -7 \\ -2 & 7 & 0 \end{bmatrix} = \begin{bmatrix} 0.5 & 2.5 & -1.7 \end{bmatrix}.$$

Thus the best strategy for player 2 is $(0, 0, 1)$.

(d)

$$\begin{bmatrix} 0 & -5 & 2 \\ 5 & 0 & -7 \\ -2 & 7 & 0 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.4 \\ 0.3 \end{bmatrix} = \begin{bmatrix} -1.4 \\ -0.6 \\ 2.2 \end{bmatrix}.$$

Thus the best strategy for player 1 is also $(0, 0, 1)$.

4. In a game, two players call out one of the numbers 1,2, or 3 simultaneously. Let S be the sum of the two numbers. If S is even, then player 2 pay S dollars to player 1. If S is odd, then player 1 pay S dollars to player 2.

   (a) Write down the payoff matrix for player 1.

   (b) Write down the payoff matrix for player 2.

   (c) Find the expected payoff of player 1 if player 1 call out the numbers 1,2,3 with probabilities 0.3,0.2,0.5 respectively, and player 2 call out the numbers 1,2,3 with probabilities 0.6,0.1,0.3 respectively.

   (d) Suppose player 2 call out the numbers 1,2,3 with probabilities 0.6,0.1,0.3 respectively. What is the best strategy for player 1 and what is his expected payoff if he uses this strategy?

Solution:

(a)

	1	2	3
1	2	-3	4
2	-3	4	-5
3	4	-5	6

(b)

	1	2	3
1	-2	3	-4
2	3	-4	5
3	-4	5	-6

(c) The expected payoff of player 1 is calculated as:

$$\begin{bmatrix} 0.3 & 0.2 & 0.5 \end{bmatrix} \begin{bmatrix} 2 & -3 & 4 \\ -3 & 4 & -5 \\ 4 & -5 & 6 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0.1 \\ 0.3 \end{bmatrix} = 1.9.$$

(d)

$$\begin{bmatrix} 2 & -3 & 4 \\ -3 & 4 & -5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0.1 \\ 0.3 \end{bmatrix} = \begin{bmatrix} 2.1 \\ -2.9 \\ 3.7 \end{bmatrix}.$$

Thus the best strategy for player 1 is $(0, 0, 1)$. And the expected payoff is 3.7.

5. Copy the following game matrices and circle all saddle points of the matrices.

(a) $\begin{pmatrix} -3 & 5 & -1 & 0 \\ -1 & -3 & 5 & -2 \\ 2 & 4 & -1 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} -3 & 5 & -3 & 0 \\ 1 & 3 & 6 & 4 \\ 0 & -4 & -1 & -3 \\ -2 & 2 & 3 & 1 \end{pmatrix}$

Solution:

(a)

					Min
	-3	5	-1	0	-3
	-1	-3	5	-2	-3
	2	4	-1	1	-1
Max	2	5	5	1	

There is no saddle point.

(b)

					Min
	-3	5	-3	0	-3
	1	3	6	4	1
	0	-4	-1	-3	-4
	-2	2	3	1	-2
Max	1	5	6	4	

The saddle point is (R_2, C_1).